Solutions to JEE Advanced Home Practice Test -2 |JEE 2024 | Paper-2

PHYSICS

SINGLE DIGIT INTEGER

1.(9) $a_x = -(3)^2 x \quad ; \qquad a_y = -(4)^2 y \; ; a_z = 0$

At $t = 0,\; x = 0,\; y = 0,\; z = 0$

$x(0) = 0 = A_1 \sin\phi_1 \;\; ; \;\; y(0) = 0 = A_2 \sin\phi_2$

$\Rightarrow \;\; \phi_1 = 0 \qquad\qquad \Rightarrow \;\; \phi_2 = 0$

At $t = 0,\; v_x = 3,\;\; v_y = 4,\;\; v_z = \frac{1}{\sqrt{2}}$

$v_x(0) = 3 = 3A_1 \cos 0 \;\; ; \;\; v_y(0) = 4 = 4A_2 \cos 0$

$A_1 = 1 \qquad\qquad A_2 = 1$

$\Rightarrow v_x(t) = 3\cos 3t \quad ; \; v_y(t) = 4\cos 4t \quad ; \quad v_z(t) = \frac{1}{\sqrt{2}}$

$$\Rightarrow \; t = \frac{\pi}{12}\sec \; ; \begin{cases} v_x = \frac{3}{\sqrt{2}} \\ v_y = 2 \;\; \Rightarrow \;\; v^2 = \frac{9}{2} + 4 + \frac{1}{2} = 9 ; \qquad k = \frac{1}{2} \times 2 \times 9 = 9\,Joule \\ v_z = \frac{1}{\sqrt{2}} \end{cases}$$

2.(5) $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{40}$

$A = A_0 e^{-\lambda t}$

$\ln\frac{A_0}{A} = \lambda t \, ; \qquad\qquad \ln\frac{A_0}{A} = 2\ln 2$

$A_0 = 2^2 A \qquad\qquad \ldots\text{(i)}$

$\lambda N_0 = 2^2 A$

$N_0 = \frac{2^2 A}{\ln 2} 40 \quad = 160 \times 10^{19} = 1.6 \times 10^{21}$

Also, $\frac{A}{A_0} = \frac{N}{N_0} \Rightarrow N = \frac{N_0}{4}$ (using equation (i)

No. of nuclei undergoing fission $= N_0 - N$

$= \frac{3}{4} N_0$

Energy released per fission $= \frac{6 \times 10^8 \times 4}{3 \times N_0} = \frac{6 \times 4 \times 10^8}{3 \times 1.6 \times 10^{21}} = 5 \times 10^{-13} J$

3.(2) $$\frac{P}{Q}=\frac{\int_0^{0.4}\rho(x)\frac{dx}{A}}{\int_{0.4}^{1}\rho(x)\frac{dx}{A}}=\frac{\left|\frac{\rho_0}{k(1-kx)}\right|_0^{0.4}}{\left|\frac{\rho_0}{k(1-kx)}\right|_{0.4}^{1}}=\frac{\frac{1}{(1-0.4k)}-\frac{1}{1}}{\frac{1}{(1-k)}-\frac{1}{(1-0.4k)}}$$

$$\Rightarrow \frac{P}{Q}=\frac{(0.4k)(1-k)}{(0.6k)}=\frac{2}{3}(1-k) \Rightarrow \frac{2}{9}=\frac{2}{3}(1-k) \Rightarrow 3k=2$$

4.(3) Thermal resistance of the rod, $R=\frac{l}{kA}$

When heat is transferred from first vessel to second, temperature of first vessel decreases while that of second vessel increases. Due to the both reasons, difference between temperature of vessels decreases. Let at an instant t, the temperature difference between two vessels be θ.

Rate of heat flowing, $H=\frac{q}{R}=\frac{KA\theta}{l}$

Amount of heat, $dQ=Hdt=\frac{KA\theta}{l}dt$ **. . . (i)**

Since gases are contained in two vessels, the process gases are undergoing are isochoric processes.

Hence, decrease in temperature of gas in first vessel, $\Delta\theta_1=\frac{dQ}{nC_v}=\frac{dQ}{2\times\frac{5R}{2}}=\frac{dQ}{5R}$

Increase in temperature of gas in second vessel is $\Delta\theta_2=\frac{dQ}{4\times\frac{3R}{2}}=\frac{dQ}{6R}$

$\therefore$ Decrease in temperature difference $(-d\theta)=\Delta\theta_1+\Delta\theta_2$;

$-d\theta=\frac{dQ}{R}\times\frac{11}{30}$ Or $-d\theta=\frac{KA\theta\times 11}{30Rl}dt$ Or $-\int_{50}^{25}\frac{d\theta}{\theta}=\frac{KA\times 11}{30lR}\int_0^1 dt$

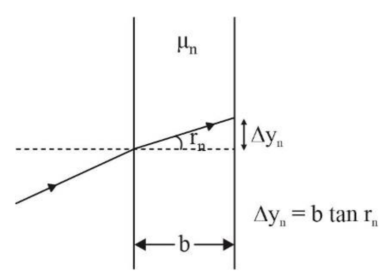
Or $\ln 2=\frac{KA\times 11}{30lR}t$ Or $t=\frac{0.693\times 30\times 242\times 10^{-2}\times 8.3\times 7}{693\times 7\times 22\times 8.3\times 10^{-4}\times 11}=3$ seconds.

5.(1) By snell's law :

$1\cdot\sin\theta=\mu_1\sin r_1=\mu_2\sin r_2=.....=\mu_n\sin r_n=......$ $\therefore \sin r_n=\frac{\sin\theta}{\mu_n}$

Also for very small, θ r_n will also be very small

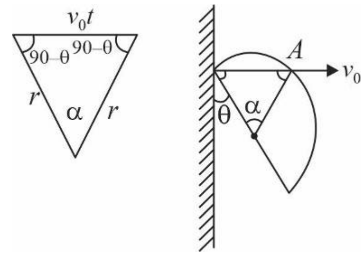
$\tan r_n=\sin r_n=r_n=\frac{\theta}{\mu_n}$ $\Rightarrow \Delta y_n=\frac{b\theta}{\mu_n}=\frac{b\theta}{\pi\left(\frac{5}{2}\right)^n}$



$\Rightarrow y=\Delta y_1+\Delta y_2+....+\Delta y_n+.....\infty$

$$\Rightarrow y=\frac{b\theta}{\pi}\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n=\frac{b\theta}{\pi}\left[\frac{\frac{2}{5}}{1-\frac{2}{5}}\right]=\frac{b\theta}{\pi}\times\frac{2}{3} \Rightarrow y=\frac{15}{\pi}\times\frac{\pi}{100}\times\frac{2}{3}cm=1mm$$

6.(4) $90-\theta+90-\theta+\alpha=180°$



$\alpha=2\theta$

$$\frac{r}{\cos\theta}=\frac{v_0 t}{\sin\alpha};\qquad \frac{\cos\theta}{r}=\frac{\sin 2\theta}{v_0 t}$$

$$\frac{\cos\theta\ v_0 t}{r}=2\sin\theta\cos\theta$$

$$2r\sin\theta=v_0 t;\qquad 2r\cos\theta\frac{d\theta}{dt}=v_0$$

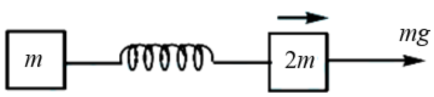
$$\frac{d\theta}{dt}=\omega=\frac{v_0}{2r\cos\theta}=\frac{v_0}{2r\times\frac{1}{2}}=\left(\frac{v_0}{r}\right)=\frac{2}{50}rad/s$$

7.(2) $T_0=2\pi\sqrt{\frac{l}{g}}$; $\quad T=\pi\sqrt{\frac{l}{g}}+\pi\sqrt{\frac{l}{g-\frac{qE}{m}}}$; $\quad \frac{qE}{mg}=\frac{200\times10^{-6}\times900}{20\times10^{-3}\times10}=\frac{9}{10}$

$$T=\pi\sqrt{\frac{l}{g}}+\pi\sqrt{\frac{l}{g\left(1-\frac{9}{10}\right)}}$$

$$T=\pi\sqrt{\frac{l}{g}}+\sqrt{10}\pi\sqrt{\frac{l}{g}}=(1+\sqrt{10})\cdot\pi\sqrt{\frac{l}{g}}\quad\Rightarrow\quad\frac{T}{T_0}=\frac{1+\sqrt{10}}{2}=\frac{4.14}{2}=2.07$$

8.(3)



$$Kx=mv\frac{dv}{dx};\qquad \frac{mv^2}{2}=\frac{kx^2}{2}$$

$$v=x\sqrt{\frac{k}{m}};\qquad mg-kx=2mv\frac{dv}{dx}$$

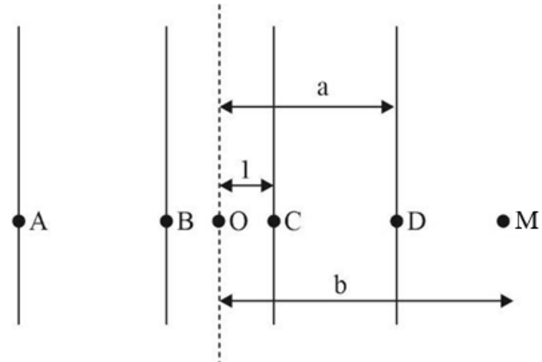
$$mgx-\frac{kx^2}{2}=\frac{2mv^2}{2};\qquad mgx-\frac{kx^2}{2}=kx^2$$

$$k\frac{3x}{2}=mg\ ;\ x=\frac{2mg}{3k}\quad=\frac{2\times3\times10}{3\times1000}=2cm\Rightarrow l=1+2=3$$

MULTIPLE CHOICE

9.(ABC) For $0<r\le1$: (Using Gauss's law)

$$2EA=\frac{2}{\varepsilon_0}\int_0^r\rho\cdot A\cdot dr=\frac{2}{\varepsilon_0}\int_0^r\rho_0 r\cdot A\cdot dr$$



$$E=\frac{\rho_0 r^2}{2\varepsilon_0}$$

For $1<r\le a$

$$2EA=\frac{2}{\varepsilon_0}\left[\int_0^1\rho_0 rAdr+\int_1^r\rho_0 r^2Adr\right]$$

$$=\frac{2}{\varepsilon_0}\left[\frac{\rho_0 A}{2}+\frac{\rho_0 Ar^3}{3}-\frac{\rho_0 A}{3}\right]$$

$$E=\frac{\rho_0}{6\varepsilon_0}(1+2r^3)$$

$$V_D-V_C=-\int_1^a \frac{\rho_0}{6\varepsilon_0}(1+2r^3)dr=-\frac{\rho_0}{6\varepsilon_0}\left|r+\frac{r^4}{2}\right|_1^a$$

$$V_D-V_C=-\frac{\rho_0}{6\varepsilon_0}\left(a+\frac{a^4}{2}-\frac{3}{2}\right)$$

For $r>a$

$$E=\frac{\rho_0}{6\varepsilon_0}(1+2a^3)$$

$$V_M-V_D=-\frac{\rho_0}{6\varepsilon_0}(1+2a^3)(b-a)$$

For $a=2$; $b=\frac{7}{2}$

$$V_M-V_D=-\frac{\rho_0}{6\varepsilon_0}(1+16)\left(\frac{7}{2}-2\right)=\frac{-17\rho_0}{4\varepsilon_0}\Rightarrow V_A-V_M=\frac{17\rho_0}{4\varepsilon_0}$$

For $a=3$; $b=5$; $\qquad E=\frac{55\rho_0}{6\varepsilon_0}$

For $a=2$; $b=3$; $\qquad V_D-V_B=-\frac{\rho_0}{6\varepsilon_0}\left(2+\frac{2^4}{2}-\frac{3}{2}\right)=-\frac{17\rho_0}{12\varepsilon_0}$

10.(ABCD) Bulb 1 : $R_1=\frac{(40)^2}{80}=20\,\Omega$

Bulb 2 : $R_2=\frac{80^2}{40}=160\,\Omega$

Bulb 3 : $R_3=\frac{40^2}{20}=80\,\Omega$

Circuit 1 :

Power P_1, P_2, P_3 respectively

$$P_1=\left(\frac{40}{180}\right)^2\times 20=0.99\,; \qquad P_2=\left(\frac{40}{180}\right)^2\times 160=7.90$$

$P_3=20W$

Circuit 2

Power P_1', P_2', P_3' respectively

$$i_3'=\frac{40}{80+\frac{160\times 20}{160+20}}=\frac{9}{22}\,; \qquad i_1'=\frac{4}{11}\ ;\ i_2'=\frac{1}{22}$$

$$P_1'=\left(\frac{4}{11}\right)^2\times 20=2.64\,W\,; \qquad P_2'=\left(\frac{1}{22}\right)^2\times 160=0.33\,W\,; \qquad P_3'=\left(\frac{9}{22}\right)^2\times 80=13.39\,W$$

11.(AB) Liquid l_1 : Adiabatic expansion $(PV^{\gamma} = \text{constant})$

$$\left(P_0 + hd_1 g + \frac{2S_1}{r}\right)\left(\frac{4}{3}\pi r^3\right)^{\gamma} = \left(P_0 + \frac{2S_1}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{\gamma}; \qquad r_1^3 = r^3\left(\frac{P_0 + hd_1 g + \frac{2S_1}{r}}{P_0 + \frac{2S_1}{r_1}}\right)^{\frac{1}{\gamma}}$$

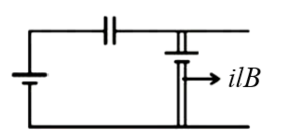
Liquid l_2 : isothermal expansion (PV = constant)

$$\left(P_0 + hd_2 g + \frac{2S_2}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_0 + \frac{2S_2}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right); \quad r_2 = r\left(\frac{P_0 + hd_2 g + \frac{2S_2}{r}}{P_0 + \frac{2S_2}{r_2}}\right)^{1/3}$$

For $d_1 = d_2 = d,\ S_1 = S_2 = S$ and $\gamma = \frac{3}{2}$; $\qquad \frac{r_2}{r_1} = \frac{\left(P_0 + hdg + \frac{2S}{r}\right)^{1\backslash 9}\left(P_0 + \frac{2S}{r_1}\right)^{2/9}}{\left(P_0 + \frac{2S}{r_2}\right)^{1/3}}$

For $d_1,\ S_1$ and $\gamma = \frac{5}{3}$: $(P^{1-\gamma}T^{\gamma} = \text{constant})$; $\qquad T_1 = T_0\left(\frac{P_0 + hd_1 g + \frac{2S_1}{r}}{P_0 + \frac{2S_1}{r_1}}\right)^{\left(\frac{1}{\gamma}-1\right)}$

12.(D) $ilB = \frac{mdv}{dt} \Rightarrow iBq = mv$



$$\varepsilon - Blv - \frac{mv}{Blc} = \frac{mR}{Bl}\frac{dv}{dt}$$

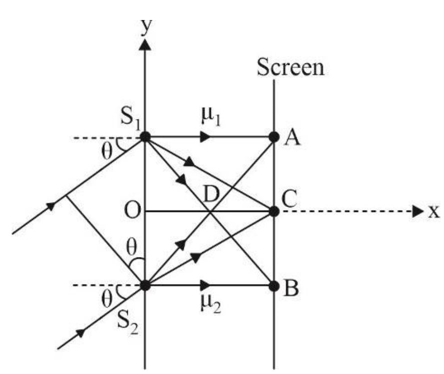
$$a = 0 \quad \Rightarrow \quad v = \frac{\varepsilon}{Bl + \frac{m}{Blc}} = \frac{\varepsilon Blc}{B^2 l^2 c + m}$$

$$q = \frac{mv}{Bl}$$

13.(BCD) At C ; optical path difference $= 2a\sin\theta + \mu_1 a\sqrt{5} - \mu_2 a\sqrt{5} = 0$

$$2a\sin\theta = (\mu_2 - \mu_1)a\sqrt{5}$$

For $\theta = 45°,\ (\mu_2 - \mu_1) = \sqrt{\frac{2}{5}}$



At A ;

$$= 2a\sin\theta + \mu(2a) - \mu(2\sqrt{2}a) = 0$$

$$\Rightarrow \mu\left(\sqrt{2} - 1\right) = \frac{1}{\sqrt{2}}; \qquad \mu = \frac{1}{\sqrt{2}\left(\sqrt{2} - 1\right)}$$

At B (for $\theta = 0^{o}$)

Path diff $= -\mu(2a) + \mu(2\sqrt{2}a) = \frac{(2n+1)\lambda}{2}$

For $\mu = \frac{\lambda}{4\left(\sqrt{2}-1\right)a}(n = 0,\ a)$ $\qquad$ Minima is obtained at B

14.(AB) $T_C = T_B\left(\frac{V_B}{V_C}\right)^{r-1} = 200\left(\frac{0.8}{0.2}\right)^{0.4} = 348°K$

Isothermal expansion $A \to B$:

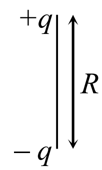
$$W_{A\to B} = nRT\ln\frac{V_B}{V_A} = 2R(200)\ln\left(\frac{0.8}{0.2}\right) = 560R$$

Adiabatic compression $B \to C$

$$W_{B\to C} = \frac{nR(T_B - T_C)}{r-1} = \frac{2R(200-348)}{\frac{7}{5}-1} = -740R$$

$$W_{A\to B\to C} = W_{A\to B} + W_{B\to C} = -180R$$

15.(B) $\vec{\tau} = I\vec{\alpha}$



$$qRE - fR = \frac{2}{3}mR^2\alpha \qquad \ldots(1)$$

$$f = ma \qquad \ldots(2)$$

$$a = R\alpha \qquad \ldots(3)$$

On solving above equations, $f = \frac{3}{5}qE$

16.(C) Stopping potential does not depend on intensity

$$\frac{hc}{2\lambda_0} - \phi_0 = 2.5 \quad \ldots(i) \qquad \frac{hc}{\lambda_0} - 3\phi_0 = 3.9 \quad \ldots(ii)$$

Solving (i) and (ii), we get $\frac{hc}{\lambda_0}$ & ϕ_0

$$\frac{hc}{\lambda_0} = 7.2\,eV\ ;\ \phi_0 = 1.1eV \qquad \Rightarrow \lambda_0 = \frac{1240}{7.2}nm = 172\,nm$$

17.(B) 1MSD $= \frac{1}{10}cm = 1\,mm$

10 VSD = 11 MSD ; 1 VSD $= \frac{11}{10}$ MSD

Least count $= \left(\frac{11}{10} - 1\right)mm = 0.1\,mm$

Zero error $= 0 + (1 - 2\times 0.1) = +0.8mm$

Attempt 1 : $3 + (1 - 3\times 0.1) - (0.8) = 2.9\,mm$

Attempt 2 : $3 + (1 - 5\times 0.1) - (0.8) = 2.7\,mm$

Mean value $= \frac{2.9 + 2.7}{2} = 2.8\,mm$

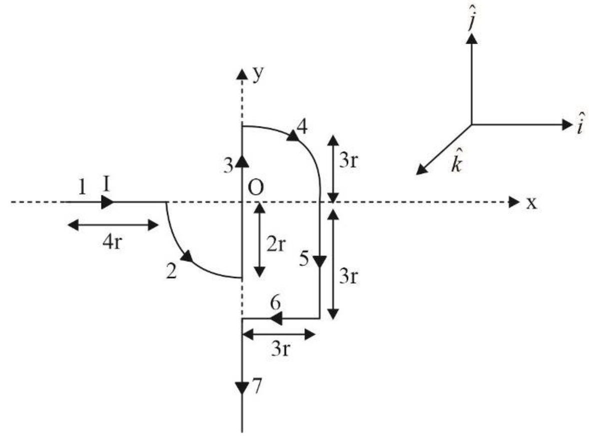
Mean absolute error $= \frac{0.1 + 0.1}{2} = 0.1\,mm$

Diameter $= (2.8 \pm 0.1)mm$

Area $= \frac{\pi d^2}{4} = \pi\frac{(2.8)^2}{4} = \pi(1.96)$; $\quad \frac{\Delta A}{A} = \frac{2\Delta d}{d} \Rightarrow \Delta A = 2\times\frac{0.1}{2.8}\times\pi\times 1.96$

$\Delta A = 0.14\pi \Rightarrow A = \pi(1.96) \pm 0.14\pi$

18.(D) $B_1 = B_7 = B_3 = 0$



$$B_2 = \frac{\mu_0 I}{4\pi(2r)}\left(\frac{\pi}{2}\right) = \frac{\mu_0 I}{16r}$$

$$B_4 = \frac{\mu_0 I}{4\pi(3r)}\left(\frac{\pi}{2}\right) = \frac{\mu_0 I}{24r}$$

$$B_5 = \frac{\mu_0 I}{4\pi(3r)}(\sin 0° + \sin 45°) = \frac{\mu_0 I}{12\sqrt{2}\pi r}$$

$$B_6 = \frac{\mu_0 I}{12\sqrt{2}\pi r}$$

$$\vec{B}_{net} = \vec{B}_2 + \vec{B}_4 + \vec{B}_5 + \vec{B}_6$$

$$\vec{B}_{net} = \frac{\mu_0 I}{16r}\hat{k} - \frac{\mu_0 I}{24r}\hat{k} - \frac{\mu_0 I}{12\sqrt{2}\pi r}\hat{k} - \frac{\mu_0 I}{12\sqrt{2}\pi r}\hat{k}$$

$$\vec{B}_{net} = \frac{\mu_0 I}{6r}\left[\frac{1}{8} - \frac{1}{\sqrt{2}\pi}\right]\hat{k}$$

Chemistry

SINGLE DIGIT INTEGER

1.(5) $CuCN \rightleftharpoons Cu^{+} + CN^{-} .... K_{sp}$

$CN^{-} + H^{+} \rightleftharpoons HCN .... \frac{1}{K_a}$

Now, $CuCN + H^{+} \rightleftharpoons HCN + Cu^{+} .... \frac{K_{sp}}{K_a} = K$

$K = \frac{1.2 \times 10^{-15}}{4.8 \times 10^{-10}} = 25 \times 10^{-7}$

At equilibrium

$CuCN + H^{+} \rightleftharpoons HCN + Cu^{+}$ $\quad ([Cu^{+}] = [HCN])$

$[H^{+}] = 10^{-3}$ as pH = 3

$K = \frac{[Cu^{+}][HCN]}{[H^{+}]}$

$25 \times 10^{-7} = \frac{s^2}{10^{-3}}$

$s^2 = 25 \times 10^{-10}$; $s = 5 \times 10^{-5} M$

Molar solubility of CuCN in buffer solution is $5 \times 10^{-5} M$

2.(9) According to Raoult's law

$\frac{\Delta P}{P_0} = \frac{P_0 - P_1}{P_0} = \frac{w_2 \times M_1}{M_2 \times w_1}$

Given,

Weight of solutes = 2g

Weight of solvent = 100 g

$\therefore \frac{\Delta P}{P_0} = \frac{2 \times 78}{M_2 \times 100}$ (Mol. Wt. of solvent = 78)

$\Rightarrow \frac{89.78 - 89.00}{89.78} = \frac{2 \times 78}{M_2 \times 100}$

Solving this we get, $M_2 = 179.56 \, g \, mol^{-1}$

90% is A by mass

$\therefore$ Amount of A in the $A_x B_y = \frac{90 \times 179.56}{100}$

No. of A atoms $= \frac{161.6}{17.96} = 9$

Wt. of B $= 179.6 - 161.6 = 18$

No. of B atoms $= \frac{18}{3.6} = 5$

$\therefore$ The molecular formula of the compound is A_9B_5

3.(1) Conductivity of saturated solution of sparingly soluble salt of $Ba_3(PO_4)_2 \Rightarrow 1.2\times10^{-5}\,ohm^{-1}\,cm^{-1}$

$\lambda^{\circ}_{eq}(BaCl_2) = 160\,ohm^{-1}\,cm^2\,eq^{-1}$

$\lambda^{\circ}_{eq}(K_3PO_4) = 140\,ohm^{-1}\,cm^2\,eq^{-1}$

$\lambda^{\circ}_{eq}(KCl) = 100\,ohm^{-1}\,cm^2\,eq^{-1}$

k_{sp} of $Ba_3(PO_4)_2 = ?$

Using the formula :

$\lambda^{\circ}_M = \lambda^{\circ}_{eq} \times$ n-factor [total sum of positive charge)

$\lambda^{\circ}_M(BaCl_2) = 160\times2 = 320 \quad [Ba^{++}]$

$\lambda^{\circ}_M(K_3PO_4) = 140\times3 = 420 \quad [3K^{+}]$

$\lambda^{\circ}_M(KCl) = 100\times1 = 100 \quad [K^{+}]$

$$\lambda^{\circ}_M[Ba_3(PO_4)_2 \Rightarrow 3\times\lambda^{\circ}_{M_{(BaCl_2)}} + 2\lambda^{\circ}_{M_{(K_3PO_4)}} - 6\times\lambda^{\circ}_{M_{(KCl)}}$$

$$\Rightarrow (3\times320)+(2\times420)-(6\times100)$$

$$\Rightarrow 960+840-600 = 1200$$

Formula : $\lambda^{\circ}_M = \dfrac{K\times1000}{S}$

$S = \dfrac{K\times1000}{\lambda^{\circ}M} \Rightarrow \dfrac{1.2\times10^{-5}\times10^{3}}{1200}$; $\quad S = 10^{-5}$

K_{sp} (in the order of 10^{-25}) : $K_{sp} = 108s^5$

$K_{sp} = 108\times(10^{-5})^5 \Rightarrow 1.08\times10^{-23}$; $\quad a = 1.08 \approx 1$

4.(4) $4NaCl + 4H_2SO_4 + MnO_2 \longrightarrow MnCl_2 + 4NaHSO_4 + 2H_2O + Cl_2(X)$

$Cl_2 + 2FeSO_4 + H_2SO_4 \longrightarrow Fe_2(SO_4)_3 + 2HCl$

2 mole 4 mole

5.(8) $XeO_3 \rightarrow sp^3$

$N(CH_3)_3 \rightarrow sp^3$

$CF_3 \rightarrow sp^3$

$PBr_5(s) \rightarrow sp^3$ (exist as PBr_4^+ in solid state)

$BeCl_2(s) \rightarrow sp^3$ (exist as polymer in solid state)

$I_3^+ \rightarrow sp^3$

$ClO_2^- \rightarrow sp^3$

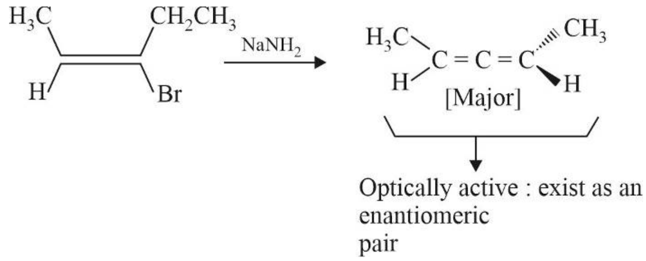
$H_2O \rightarrow sp^3$

$SnCl_2 \rightarrow sp^2$

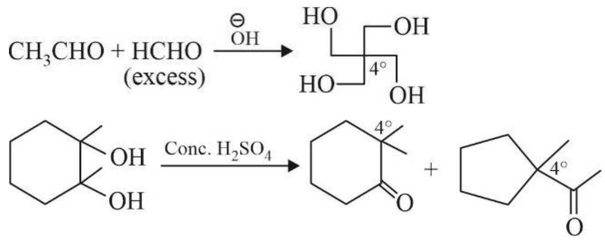
$Ca\underline{C}O_3 \rightarrow sp^2$

$N(SiH_3)_3,\ O(SiH_3)_2 \rightarrow sp^2$ (Due to back bonding)

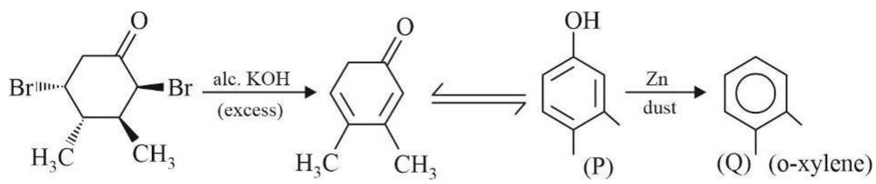
6.(2)



7.(3)



8.(3)



$$\underset{(Q)}{o-\text{xylene}} \xrightarrow{O_3,\ Zn/H_2O} CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CHO + \underset{\underset{\displaystyle CHO}{|}}{CHO} + CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$$

MULTIPLE CHOICE

9.(BC) $C_2H_5OH + O_2 \longrightarrow CH_3COOH + H_2O$

$$d = \frac{m}{V}$$

100 mL wine sample $\rightarrow 8.5\,\text{mL}\ C_2H_5OH$

1 mL wine sample $\rightarrow \frac{8.5}{100}\,\text{mL}\,C_2H_5OH$

$m = d \times V$; $\quad m = 0.82\,\text{g}\,\text{mL}^{-1} \times \frac{8.5}{100}\,\text{mL} = 0.0697\ \text{g}$

$$n_{C_2H_5OH} = \frac{0.0697}{46} = 0.0015\,\text{mol}$$

Acc. to balanced chemical equation

$$n_{CH_3COOH} = 0.0015\,\text{mol}$$

$$\%\ \text{Purity} = \frac{0.024}{0.0015 \times 60} \times 100\% = 26.66\%$$

1 mL wine sample $\rightarrow \frac{8.5}{100}\,\text{mL}\ C_2H_5OH$

200 mL wine sample $\rightarrow \frac{8.5}{100} \times 200\,\text{mL}\,C_2H_5OH$

$$\rightarrow 8.5 \times 2\,\text{mL } C_2H_5OH$$

$$\rightarrow 17\,\text{mL } C_2H_5OH$$

$$m = d \times V \qquad = 0.82\,\text{g mL}^{-1} \times 17\,\text{mL} = 13.94\text{g}$$

10.(ABC) Net cell reaction is

$$Cd(s) + 2AgCl(s) \rightleftharpoons 2Ag(s) + Cd^{2+}(aq) + 2Cl^-(aq)$$

$$\Delta G^\circ_{50^\circ C} = -nFE^\circ = -2 \times 96500 \times 0.6 = -115800\,J$$

$$\Delta G^\circ_{0^\circ C} = -nFE^\circ = -2 \times 96500 \times 0.7 = -135100\,J$$

$$\Delta S^\circ = nF \cdot \left(\frac{E_2^\circ - E_1^\circ}{T_2 - T_1}\right) = 2 \times 96500 \times \frac{0.6 - 0.7}{50} = -386\,J/K$$

$$\Delta H^\circ = \Delta G^\circ + T \cdot \Delta S^\circ$$
$$= (-135100) + 273 \times (-386) = -240478\,J$$

11.(D) Theory based

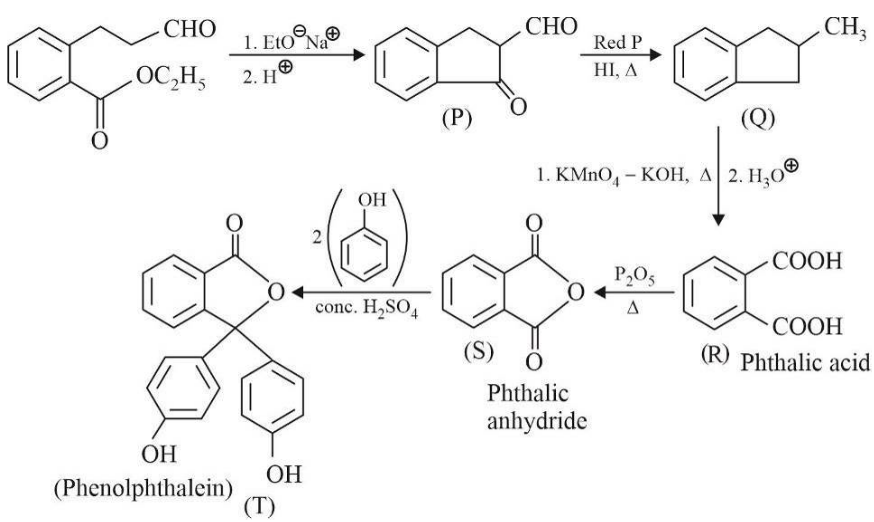
12.(BCD) Molten matte is not electrolysed

Solidified copper obtained has blister appearance due to evolution of SO_2

$$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$$

SiO_2 is used as a flux to remove impurity

13.(AD)



14.(BD) In polymerization of ethylene to polyethylene Zeigler natta $TiCl_4 + Al(C_2H_5)_3$ catalyst is used

PHBV is obtained by polymerization of 3-hydroxybutanoicacid and 3-hydroxypentanoic acid which undergoes bacterial degradation in the environment

SINGLE CHOICE

15.(A) Unit cell of CCP is FCC, therefore,

Number of oxide ions = 4

Number of tetrahedral voids $= 2 \times 4 = 8$

Number of octahedral voids = 4

According to formula,

Number of cations X present per unit cell = 1

Number of cations Y present per unit cell = 2

Since,

Y are equally distributed in octahedral voids and tetrahedral voids i.e.,

One Y cation in octahedral and other Y cation in tetrahedral void

$\therefore$ Total number of octahedral voids captured = 1 + 1 = 2

X is also present in octahedral void

$\Rightarrow$ Fraction of octahedral voids occupied $= \frac{2}{4} = \frac{1}{2}$

16.(B) $\underset{(X)}{PCl_5} + CH_3COOH \longrightarrow CH_3COCl + \underset{(Y)}{POCl_3} + HCl$

$\underset{(Y)}{POCl_3} + 3H_2O \longrightarrow \underset{(Z)}{H_3PO_4} + 3HCl$

$4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$

17.(D) $\underset{\text{Yellow colour}}{SnS_2} \downarrow + 4HCl \longrightarrow SnCl_4 + 2H_2S \uparrow$

$Cu(NO_3)_2 + 2NaOH \longrightarrow \underset{\text{Blue colour}}{Cu(OH)_2} + 2NaNO_3$

$Pb(NO_3)_2 + 2HCl \longrightarrow PbCl_2 \downarrow + 2HNO_3$

$Bi(OH)_3 + 3HCl \longrightarrow BiCl_3 + 3H_2O$

18.(D) Seliwanoff's reagent (Resorcinol + conc.HCl)

Ketoses $\xrightarrow{\text{SR}}$ deep cherry colour

Aldoses $\xrightarrow{\text{SR}}$ Faint pink

Mathematics

SINGLE DIGIT INTEGER

1.(1) $\dfrac{\cos\theta_1}{\cos\theta_2}+\dfrac{\sin\theta_1}{\sin\theta_2}=1 \Rightarrow \sin(\theta_1+\theta_2)=\sin\theta_2\cos\theta_2$

Similarly $\sin(\theta_0+\theta_2)=\sin\theta_2\cos\theta_2$

$$\Rightarrow \sin(\theta_1+\theta_2)-\sin(\theta_0+\theta_2)=0 \Rightarrow 2\cos\left(\frac{\theta_0+\theta_1+2\theta_2}{2}\right)\cdot\sin\left(\frac{\theta_1-\theta_0}{2}\right)=0$$

$$\Rightarrow \frac{\theta_0+\theta_1+2\theta_2}{2}=(2n+1)\frac{\pi}{2},\ n\in I$$

$$\because\ \sin\left(\frac{\theta_1-\theta_0}{2}\right)\neq 0$$

Now, $\left(\dfrac{\cos\theta_1}{\cos\theta_2}+\dfrac{\sin\theta_1}{\sin\theta_2}\right)\left(\dfrac{\cos\theta_0}{\cos\theta_2}+\dfrac{\sin\theta_0}{\sin\theta_2}\right)=1\times1=1$

$$\Rightarrow \left(\frac{\cos\theta_1\cos\theta_0}{\cos^2\theta_2}+\frac{\sin\theta_1\sin\theta_0}{\sin^2\theta_2}\right)+\frac{\sin\theta_1\cos\theta_0+\cos\theta_1\sin\theta_0}{\sin\theta_2\cos\theta_2}=1$$

$$\Rightarrow \left(\frac{\cos\theta_1\cos\theta_0}{\cos^2\theta_2}+\frac{\sin\theta_1\sin\theta_0}{\sin^2\theta_2}\right)+\frac{2\sin\left(\theta_0+\theta_1\right)}{\sin 2\theta_2}=1$$

$$\Rightarrow \left(\frac{\cos\theta_1\cos\theta_0}{\cos^2\theta_2}+\frac{\sin\theta_1\sin\theta_0}{\sin^2\theta_2}\right)+\frac{2\sin\left(2n\pi+\pi-2\theta_2\right)}{\sin 2\theta_2}=1$$

$$\Rightarrow \left(\frac{\cos\theta_1\cos\theta_0}{\cos^2\theta_2}+\frac{\sin\theta_1\sin\theta_0}{\sin^2\theta_2}\right)+2=1$$

2.(1) $(1+ye^x)(e^x dy+ye^x dx)=2x dx$

$$\Rightarrow \frac{(1+ye^x)^2}{2}=x^2+C \qquad \Leftarrow y(0)=1$$

$$\Rightarrow ye^x+1=\sqrt{2x^2+4} \qquad \therefore\ g(-1)=\left(\sqrt{6}-1\right)e$$

3.(0) $f(x)=(1-x^9)^{\frac{1}{7}}=y \Rightarrow 1-x^9=y^7 \Rightarrow x=(1-y^7)^{\frac{1}{9}}$

$$\Rightarrow f^{-1}(x)=(1-x^7)^{\frac{1}{9}}$$

Clearly $\displaystyle\int_a^b f(x)\,dx+\int_{f(a)}^{f(b)} f^{-1}(x)dx=bf(b)-af(a)$

$$\Rightarrow I_0=\int_0^1 (1-x^9)^{\frac{1}{7}}dx+\int_1^0 (1-x^7)^{\frac{1}{9}}dx$$

$$=1.0-0.1=0$$

4.(5) We have $y = \log_2\left(\frac{(x^2+x+1)(x^2-x+1)}{x^4+x^3+2x^2+x+1}\right) = \log_2\left(\frac{x^2-x+1}{x^2+1}\right)$

Assume $z = \frac{x^2-x+1}{x^2+1}$

$(z-1)x^2 + x + (z-1) = 0$ as $x \in R$, so

$D \geq 0 \Rightarrow 1 - 4(z-1)^2 \geq 0 \Rightarrow (z-1)^2 \leq \frac{1}{4} \Rightarrow \frac{1}{2} \leq z \leq \frac{3}{2}$

So, $y_{\min} = \log_2\left(\frac{1}{2}\right) = -1$ & $y_{\max} = \log_2\left(\frac{3}{2}\right) = (\log_2 3 - 1)$

Hence $y_{\min} + y_{\max} = \log_2 3 - 2 \equiv ((\log_2 m) - n)$

$\therefore\ m = 3, n = 2$

Hence, $(m+n) = 3+2 = 5$

5.(5) $l = \lim_{x\to\infty} x \ln\left(\frac{e(1+\left(\frac{1}{x}\right))}{\left(1+\left(\frac{1}{x}\right)\right)^x}\right)$ $(\infty \times 0 \text{ form})$

$= \lim_{x\to\infty} x\left(1 + \ln\left(1+\frac{1}{x}\right) - x\ln\left(1+\frac{1}{x}\right)\right)$

Put $x = \frac{1}{t}$; $x \to \infty, t \to 0$

Hence, $I = \lim_{t\to 0}\frac{1}{t}\left(1 + \ln(1+t) - \frac{\ln(1+t)}{t}\right)$

$= \lim_{t\to 0}\left[\frac{\ln(1+t)}{t} + \frac{t - \ln(1+t)}{t^2}\right] = \lim_{t\to 0}\frac{\ln(1+t)}{t} + \frac{\left(t - t + \frac{t^2}{2} - \frac{t^3}{3} + ...\infty\right)}{t^2}$

$= 1 + \frac{1}{2} = \frac{3}{2} = \frac{m}{n}$

$\Rightarrow\ (m+n) = 5$

6.(2) $I = A + (A^T)^2 = A + (A^2)^T$

$\Rightarrow\ I^T = A^T + A^2 \ \Rightarrow\ A^T = I - A^2$

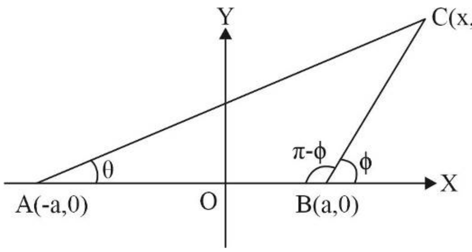
$I = A + (I - A^2)^2 \ \Rightarrow\ I = A + I + A^4 - 2A^2$

$\Rightarrow\ A^4 + A = 2A^2 \ \Rightarrow\ A^3 + I = 2A \ \Rightarrow\ K = 2$

7.(2) Choose AB for x-axis and $A \equiv (-a,0)$ & $B \equiv (a,0)$. If $\angle A = \theta$ & $\angle B = \pi - \phi$ the given condition is $\pi - \phi = 2\theta$ or $\tan\phi = -\tan 2\theta$, the locus of C is given by

$$\frac{y}{x-a} = -\frac{2\frac{y}{x+a}}{1-\left(\frac{y}{x+a}\right)^2}$$

Which simplifies to $3x^2 + 2xa - y^2 = a^2$ and this can be written in the form $3\left(x+\frac{a}{3}\right)^2 - y^2 = \frac{4a^2}{3}$



The locus is a hyperbola with centre at $\left(-\frac{a}{3},0\right)$ and transverse axis along the x-axis i.e, the base AB**.**

As $(-a,0)$ satisfies the equation the point A is one of the vertices

Eccentricity $= \sqrt{\frac{\left(\frac{4a^2}{9}+\frac{4a^2}{3}\right)}{\frac{4a^2}{9}}} = 2$

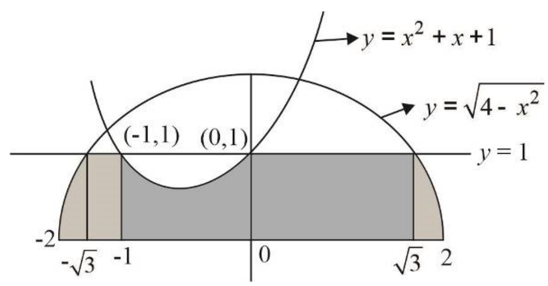
8.(7) As $-2 \le x < 2$

$1 \le \sin^2\frac{x}{4} + \cos\frac{x}{4} < 2 \Rightarrow y \le 1$

Required area

$$2\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx + \left(\sqrt{3}-1\right) + \int_{-1}^{0}(x^2+x+1)\,dx + \sqrt{3}$$

$$= \frac{4\pi-1}{6} + \sqrt{3} \text{ sq. units} \Rightarrow a+b-c = 7$$



MULTIPLE CHOICE

9.(BCD) The fact that the two circumcircles are congruent means the chord AD must subtend the same angle in both the circles.

$\angle ABC = \angle ACB \quad \Rightarrow \Delta ABC$ is isosceles

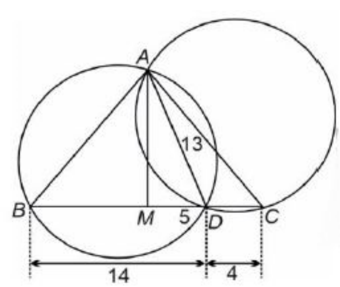
Now, AM is the altitude of ΔABC

$AM = 12 \Rightarrow$ Area $= \frac{18 \cdot 12}{2} = 108 \quad (\Delta = \frac{1}{2}$ (base) (altitude))

Also $\tan B = \tan C = \frac{12}{9}$

$\Rightarrow B = \tan^{-1}\left(\frac{4}{3}\right) \qquad \therefore \angle A = \pi - 2\tan^{-1}\left(\frac{4}{3}\right)$

$= \pi - \left[\pi + \tan^{-1}\frac{2\cdot(4/3)}{1-(16/9)}\right] = \tan^{-1}\left(\frac{24}{7}\right)$



10.(ABCD) $y = x^{1/3}(x-1)$; $\quad \dfrac{dy}{dx} = \dfrac{4}{3}x^{1/3} - \dfrac{1}{3}\cdot\dfrac{1}{x^{2/3}} = \dfrac{1}{3x^{2/3}}[4x-1]$

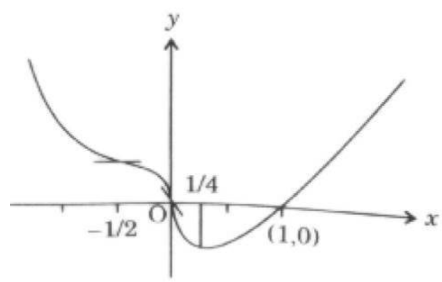
Hence, f is $\uparrow$ for $x > \dfrac{1}{4}$ & $f\downarrow$ for $x < \dfrac{1}{4}$

$x^{2/3}$ is always positive and $x = \dfrac{1}{4}$ the curves has a local minima

Now, $f'(x) = \dfrac{4}{3}x^{1/3} - \dfrac{1}{3}x^{-2/3}$ (Non-existent at $x = 0$, vertical tangent)

$f''(x) = \dfrac{4}{9}\cdot\dfrac{1}{x^{2/3}} + \dfrac{1}{3}\cdot\dfrac{2}{3}\cdot\dfrac{1}{x^{5/3}} = \dfrac{2(2x+1)}{9x^{5/3}}$ (Inflection point)

Graph of $f(x)$ is as



$$A = \int_0^1 (x^{4/3} - x^{1/3})dx = \frac{3}{7}x^{7/3} - \frac{3}{4}x^{4/3}\Big]_0^1 = \left|\frac{3}{7} - \frac{3}{4}\right| = 3\left|\frac{4-7}{28}\right| = \frac{9}{28} \text{ sq units}$$

11.(ABD) $x^2\dfrac{dy}{dx} = y^2 e^{1/x} \Rightarrow \dfrac{dy}{y^2} = \dfrac{e^{1/x}}{x^2}dx$

Integrating both sides

$\dfrac{-1}{y} = \displaystyle\int \dfrac{e^{1/x}}{x^2}dx + C \qquad \Rightarrow$ Putting $\dfrac{1}{x} = t$

$\Rightarrow \dfrac{-1}{x^2}dx = dt$; $\qquad \dfrac{-1}{y} = -\displaystyle\int e^t dt + C$

$\Rightarrow \dfrac{-1}{y} = -e^t + C \qquad \Rightarrow \dfrac{-1}{y} = -e^{1/x} + C$

$\because \displaystyle\lim_{x\to 0^-} f(x) = 1 \qquad \Rightarrow -1 = 0 + C \Rightarrow C = -1$

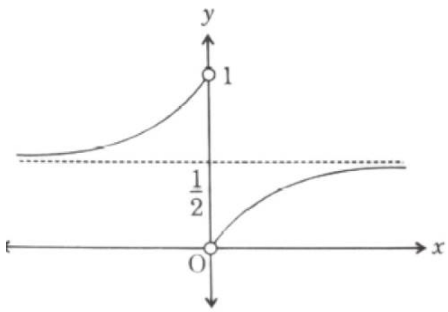
$\dfrac{-1}{y} = -e^{1/x} - 1 \qquad \Rightarrow y = \dfrac{1}{1+e^{1/x}} \Rightarrow \dfrac{dy}{dx} = \dfrac{-1}{(1+e^{1/x})^2}e^{1/x}\left(\dfrac{-1}{x^2}\right) = \dfrac{e^{1/x}}{x^2(1+e^{1/x})^2}$

$\therefore \dfrac{dy}{dx} > 0 \ \forall x \in R - \{0\}$

$\displaystyle\lim_{x\to\pm\infty} \frac{1}{1+e^{1/x}} = \frac{1}{2}$

$\displaystyle\lim_{x\to 0^+} \frac{1}{1+e^{1/x}} = 0$

$\therefore$ Graph of the function is given below :



From the graph options (A), (B) and (D) are correct

12.(BCD) $\overrightarrow{AB} = \bar{b} - \bar{a} = 4\hat{i} - 12\hat{j} - 3\hat{k}$

$\overrightarrow{CD} = \bar{d} - \bar{c} = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\overrightarrow{AC} = \bar{c} - \bar{a} = 3\hat{i} - 7\hat{j} + 2\hat{k}$

$\overrightarrow{BD} = \bar{d} - \bar{b} = 2\hat{i} + 9\hat{j} - 7\hat{k}$

By definition $d = \dfrac{(\overrightarrow{AB} \times \overrightarrow{CD}) \cdot \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ …(i)

$= \dfrac{(\overrightarrow{AB} \times \overrightarrow{CD}) \cdot \overrightarrow{BD}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ …(ii)

$\overrightarrow{AB} \times \overrightarrow{CD} = 13(12\hat{i} + 3\hat{j} + 4\hat{k})$

$\therefore \left|\overrightarrow{AB} \times \overrightarrow{CD}\right| = 169$

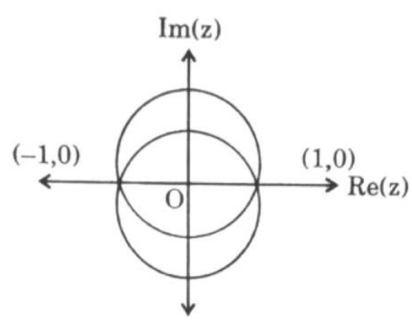
$\therefore \quad d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (3\hat{i} - 7\hat{j} + 2\hat{k})$

$= \dfrac{23}{13}$ [Using equation (1)]

Also $d = \dfrac{13(12\hat{i} + 3\hat{j} + 4\hat{k})}{169} \cdot (2\hat{i} + 9\hat{j} - 7\hat{k})$

$= \dfrac{23}{13}$ [Using equation (ii)]

13.(BCD) We have $\left|z + \dfrac{1}{z}\right| = 2$



$\Rightarrow \left|z + \dfrac{1}{z}\right|^2 = 4 \Rightarrow \left(z + \dfrac{1}{z}\right)\left(\bar{z} + \dfrac{1}{\bar{z}}\right) = 4$

$\Rightarrow z\bar{z} + \dfrac{\bar{z}}{z} + \dfrac{z}{\bar{z}} + \dfrac{1}{z\bar{z}} = 4 \Rightarrow z\bar{z} + \dfrac{\bar{z}^2 + z^2}{z\bar{z}} + \dfrac{1}{z\bar{z}} = 4$

$\Rightarrow (z\bar{z})^2 - 4(z\bar{z}) + z^2 + \bar{z}^2 + 1 = 0$

$\Rightarrow (z\bar{z})^2 - 2z\bar{z} + 1 - 2z\bar{z} + z^2 + \bar{z}^2 = 0$

$\Rightarrow \quad (z\bar{z}-1)^2+(z-\bar{z})^2=0$

$\Rightarrow \quad (z\bar{z}-1)^2-i^2(z-\bar{z})^2=0$

$\Rightarrow \quad (z\bar{z}-1+i(z-\bar{z})(z\bar{z}-1-i(z-\bar{z})=0$

Each of the above factors represents a circle with centre and radius (in ordered pair) as $(i,\sqrt{i\bar{i}+1})$ and $(-i,\ \sqrt{i\bar{i}+1})$ i.e., $\left((0,1),\ \sqrt{2}\right)$ and $\left((0,-1),\ \sqrt{2}\right)$ respectively

The union of the circle with equations

$x^2+y^2-2y-1=0$ & $x^2+y^2+2y-1=0$

These circles cut each other orthogonally.

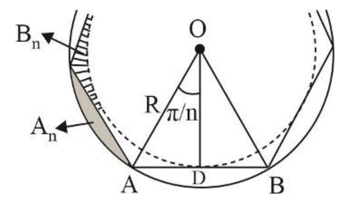
14.(BC) In ΔOAD

$OD - R\cos\frac{\pi}{n},\ AD=R\sin\frac{\pi}{n}$

$A_n =$ Area of circle (circumscribing polygon) – Area of polygon

$$A_n=\pi R^2-\frac{n}{2}R^2\sin\left(\frac{2\pi}{n}\right)$$

$B_n =$ Area of polygon – Area of circle (Inscribed)



$$B_n=\frac{n}{2}R^2\sin\left(\frac{2\pi}{n}\right)-\pi R^2\cos^2\left(\frac{\pi}{n}\right)$$

If $n=6$ then

$$A_n=\pi R^2-\frac{3\sqrt{3}}{2}R^2$$

If $n = 4$ then value of

$$B_n=2R^2-\frac{\pi R^2}{2}=R^2\left(\frac{4-\pi}{2}\right)$$

SINGLE CHOICE

15.(A) $\lim\limits_{n\to\infty}\dfrac{1^2n+2^2(n-1)+3^2(n-2)+....+n^2(n-(n-1))}{\Sigma n^3}$

$N^r=n(1^2+2^2+....+n^2)-(1.2^2+2.3^2+3.4^2+...+(n-1)n^2)$

$$=n\Sigma n^2-\sum_{r=2}^{n}(r-1)\cdot r^2=n\Sigma n^2-\sum_{r=1}^{n}(r^3-r^2)$$

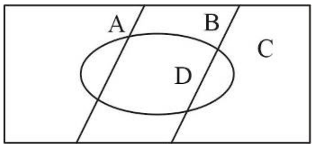
$$=n\Sigma n^2-\left[\Sigma n^3-\Sigma n^2\right]=(n+1)\Sigma n^2-\Sigma n^3$$

$$\therefore\ l=\lim_{n\to\infty}\frac{(n+1)\Sigma n^2-\Sigma n^3}{\Sigma n^3}$$

$$=\lim_{n\to\infty}\frac{4(n+1)n(n+1)(2n+1)}{6n(n+1)n(n+1)}-1=\frac{4}{3}-1=\frac{1}{3}$$

16.(C) $P\left(\frac{C}{D}\right)=\frac{P(C\cap D)}{P(D)}$

$$P\left(\frac{C}{D}\right)=\frac{P(C)\cdot P\left(\frac{D}{C}\right)}{P(D)} \qquad \ldots(i)$$



$P(D)=P(A\cap D)+P(B\cap D)+P(C\cap D)$

$=P(A)\cdot P\left(\frac{D}{A}\right)+P(B)\cdot P\left(\frac{D}{B}\right)+P(C)\cdot P\left(\frac{D}{C}\right)$

$=(0.35)(0.02)+(0.25)(0.01)+(0.40)(0.03)=0.0070+0.0025+0.0120=0.0215$

$P\left(\frac{C}{D}\right)=\frac{0.0120}{0.0215}=\frac{120}{215}=\frac{24}{43}$

17.(D) $X^2=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}a^2+bc & ab+bd\\ ac+cd & bc+d^2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

They should be simultaneously satisfied

$a^2+bc=1 \qquad \ldots(i)$

$b(a+d)=0 \qquad \ldots(ii)$

$c(a+d)=0 \qquad \ldots(iii)$

$bc+d^2=1 \qquad \ldots(iv)$

From equation (i)

$bc=1-a^2$

From equation (iv)

$1-a^2+d^2=1$

$a^2-d^2=0$

$a=d \text{ or } a=-d$

Case-I : $a=d$

From equation (ii) and (iii), $2ab=0$ & $2ac=0$

If $a=0$ then $d=0$ and from equation (i) $c=\frac{1}{b}$

Hence, general matrix X satisfying $X^2=I$ can be $\begin{pmatrix}0 & b\\ \frac{1}{b} & 0\end{pmatrix}$

$\Rightarrow$ Infinite in number with $b\in R-\{0\}$

18.(B) Case-1 : *abbc*

We can choose a in 9 ways, *b* in 9 ways and *c* in 9 ways. Hence, there are 9^3 possibilities for this case

Case-2 : *bbac*

There are 9 possibilities for *b*, and 9 possibilities for each *a* and *c*. Once again, we have that there are 9^3 possibilities for this case.

Case-3 : *acbb*

Once again, there are 9 possibilities for *a* and *c*, and 9 possibilities for *b*. Hence there are 9^3 possibilities for this case.

Case-4 : *aabb*

We have 9 possibilities for *a* and 9 possibilities for *b*. We thus have 81 possibilities for this case.

We can now sum up our answers, to get a total of $3.\,9^3 + 81 = 2268$ total ways.